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Comparison of transmission loss prediction using condensed equivalent plate models

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Abstract

Equivalent plate models are commonly used in order to simulate multilayer system as one single layer. This kind of approach enables for instance to reduce the mesh size and thus the number of degrees of freedom in finite element models. Usual condensed model rely on thin plate theory and are obviously limited when considering thick multilayers. In this work we present a comparison of the transmission loss prediction using different analytical models of solid layers: thin plate, thick plate (first order shear deformation) and 3D modelling (3D linear elasticity) . Doing so, analytical limit for the validity of the thin plate theory is derived from wavenumber analyses. Applications to industry inspired cases are presented.

Keywords: Equivalent plate model, Transmission loss

1 INTRODUCTION

In recent years, multi-layered partitions have been used widely for better sound comfort and noise attenuation. Advanced composite structures are one type of multi-layered systems which are being used increasingly in various industries such as aerospace and aircraft industries. Sandwich composites which exhibit high stiffness with light weight are widely used in the transportation and construction industries. Since the interactions between the different layered materials affect the acoustic performance of the multi-layered system, it is not always optimized by including only the best materials. Due to the diversity of materials used in the multi-layered system, modelling of this system often requires suitable mesh types for the material used and increases the total mesh count in the Finite Element modelling. Additionally, it would lead to high computation time due to these complexities. Therefore, in many engineering cases, it is of high interest to condense the behaviour of a multi-layer system to a single layer material. This aims at reducing the mesh size of the finite element model which will lead to less computation time. Guyader and Cacciolati[1, 2] had developed an equivalent thin plate model for the multi-layer plate made of isotropic layers. Ross, Kerwin and Ungar (RKU) [3–6] developed a simplified dynamic model for a three layer system to find the equivalent material properties. The model is initially developed for beams which was later used for plates as well. For the multi-layer plate consists of general orthotropic layers, Woodcock[7] developed a vibroacoustic model which was later reformulated by Loredo and Castel[8] for anisotropic layers. Marchetti, Ege, and Leclere[9] recently developed an equivalent plate model for multi-layer thin plates made of orthotropic layers.

The goal of equivalent plate models is to find a set of parameters for a thin plate model which embeds the bending and shear behaviours of a multilayer. The parameters are computed at each frequency and used with a thin plate model. As the compressional effects are not taken into account in thin or thick plate models, this requires a frequency limit to safely apply these models to have only equivalent bending effect. In this direction, the present paper compares the elastic thin (Love-Kirchoff), thick (Reissner-Mindlin) and solid (linear elasticity) plate models to find the analytical frequency limit to apply the classical plate theory to find the transmission loss across the plate.

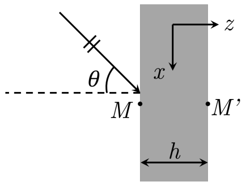

Figure 1. Plane wave incidence on the elastic medium

2 VIBROACOUSTIC MODEL FOR THE ELASTIC PLATES

2.1 Transfer matrix for elastic solid

Generally, the Transfer Matrix Method (TMM) is used to compute the vibroacoustic indicators of the multi-layer system from the transfer matrices of the stratified media. In the present work, this method is adopted to compute the transmission loss of a single layer of isotropic elastic medium. Consider an infinite layer along the x and y directions with thickness h along the z direction as shown in the Fig. 1, excited by a plane wave with incidence angle θ. The layer has z axis origin at the middle surface as a reference. By considering both longitudinal and shear wave propagation, the associated displacements and stresses can be obtained from theory of elasticity approach and the transfer matrix ($[T^{ES}]$) which relates the state variables (V^{ES}) between the points M and M' is written as follows:

$$V^{ES}(M) = [T^{ES}]_{4\times4} V^{ES}(M') \tag{1}$$

The elements of the matrix $[T^{ES}]$ are given in the book by Allard and Atalla[10] and the state vector (V^{ES}) which describes the acoustic field is given as:

$$V^{ES} = \{\dot{u}_x, \dot{u}_z, \sigma_{zz}, \sigma_{xz}\}^T \tag{2}$$

In the above equation, $\dot{u}$ and σ are the velocities and stresses, respectively. The transmission loss of the elastic layer can be computed by Transfer Matrix Method (TMM), for example, as detailed by Allard and Atalla[10]. As the acoustic field in the elastic layer is completely described by the longitudinal and shear waves, the TL solution obtained from $[T^{ES}]$ is taken as a reference to compare the other vibroacoustic models. Similarly for the following plate models, by having appropriate transfer matrix, the TL is computed in the same way.

2.2 Transfer matrix for plate theories

If the acoustic field in the elastic layer (Fig. 1) is described by the state vector $V^P = \{\sigma_{zz}, \dot{u}_z\}^T$, then the general form the transfer matrix for the plate theories can be expressed by the following equation:

$$V^P(M) = [T^P]V^P(M') = \begin{bmatrix} 1 & -Z_P \\ 0 & 1 \end{bmatrix} V^P(M') \tag{3}$$

Here, Z_P is the mechanical impedance of the plate and it is expressed as below based on the theory adopted.

$$Z_P = \begin{cases} Z_{thin} \\ Z_{thick} \end{cases} = \begin{cases} j\omega m_s \left(1 - \frac{Dk_t^4}{\omega^2 m_s}\right) \\ \frac{-j}{\omega\left(1+\frac{k_t^2 D - I_z\omega^2}{G^* h}\right)} \left(k_t^4 D - m_s\omega^2 + \left(\frac{I_z m_s}{G^* h}\omega^2 - k_t^2 \frac{Dm_s}{G^* h}\right)\omega^2 - k_t^2 I_z \omega^2\right) \end{cases} \tag{4}$$

where, $k_t = k_0 \sin\theta$ with k_0 as the wave number in the air, m_s is the mass density per unit area, D is the bending stiffness, $G^* = G\xi$ with G as the shear modulus of the plate, ξ is the shear correction factor, $I_z = \frac{m_s h^2}{12}$ is the mass moment of inertia, and ω is the circular frequency. The mechanical impedances of the thin and thick plates are obtained from the elastic plate theories of Love-Kirchoff[11] and Reissner-Mindlin[12, 13] respectively.

2.3 Transfer matrix for equivalent thin plate theories

2.3.1 Guyader model

By applying classical plate theory to the multi-layer viscoelastic plate, Guyader and Cacciolati [1, 2] developed an analytical model to determine the equivalent complex bending stiffness as a function of the excitation frequency. In this model, each layer is assumed to have Reissner-Mindlin type displacements or in other words each layer is modeled by considering bending, membrane and shear effects. Displacement and shear stress continuity conditions are applied to obtain the equations of motion of the multi-layer plate which are interestingly expressed as a function of only the first layer displacement field. Finally, assuming that the transverse displacement is the same for both the equivalent single layer and the multi-layer plate, this model determines the equivalent bending stiffness by using Love-Kirchhoff thin plate theory.

2.3.2 RKU model

Ross, Kerwin and Ungar (RKU) [3–6] developed a vibroacoustic model for a three layered plate by assuming the second layer (core) to be soft in nature compared to the other two stiff layers (skins), and the core is mainly stressed in shear. Expressions for the equivalent thin plate properties such as bending stiffness and loss factor are derived as a function of the shear parameter that describes how well the viscoelastic layer couples flexural motions of the other two layers. The complete derivation can be found in the articles [3–6] as well as in the book by Beranek and Ver[14].

2.3.3 Added Stiffness (AS) Model

This model is based on computing the equivalent properties by adding the respective individual layer properties. The bending stiffness of each layer is obtained with respect to the neutral axis of the multi-layer plate. One can note that a similar approach is often used for composite materials as described in [15] for beams.

After obtaining the equivalent bending stiffness (D_{eq}) from the above equivalent thin plate theories, it can be substituted in Z_{thin} for the TL computation. It must be noted that D_{eq} is a dynamic bending stiffness (i.e, function of the frequency) for the equivalent plate models.

3 COMPARISON WITH AS MODEL: A MULTI-LAYER PLATE EXAMPLE

In this section, a multi-layer plate (Plasterboard/Glue/Plasterboard) of practical application is taken to illustrate the necessity of the frequency limit, for the AS model, to be found. The material properties are listed in the Table 1. The TL curves pertaining to the diffuse field excitation are generated from the AS model (equivalent

Table 1. Material properties

Material parameters	Units	Plasterboard	Glue	Concrete
Thickness	mm	12.5	0.5	140
Density	kg/m^3	700	600	2300
Young's modulus	GPa	3	0.002	30
Loss factor	-	0.02	1	0.02
Poisson's ratio	-	0.3	0.3	0.3

thin plate model) and compared with linear elasticity solution in the Fig. 2. It can be observed that AS model starts to differ from the reference solution before the critical frequency (though it predicts the critical frequency with good accuracy) of the multi-layer plate. It also shows that the stiffness is predicted with a fair accuracy while the loss factors is largely underestimated. Furthermore, at higher frequencies, the difference in TL between the reference solution and equivalent plate models is greater than 4 dB. This example the need for

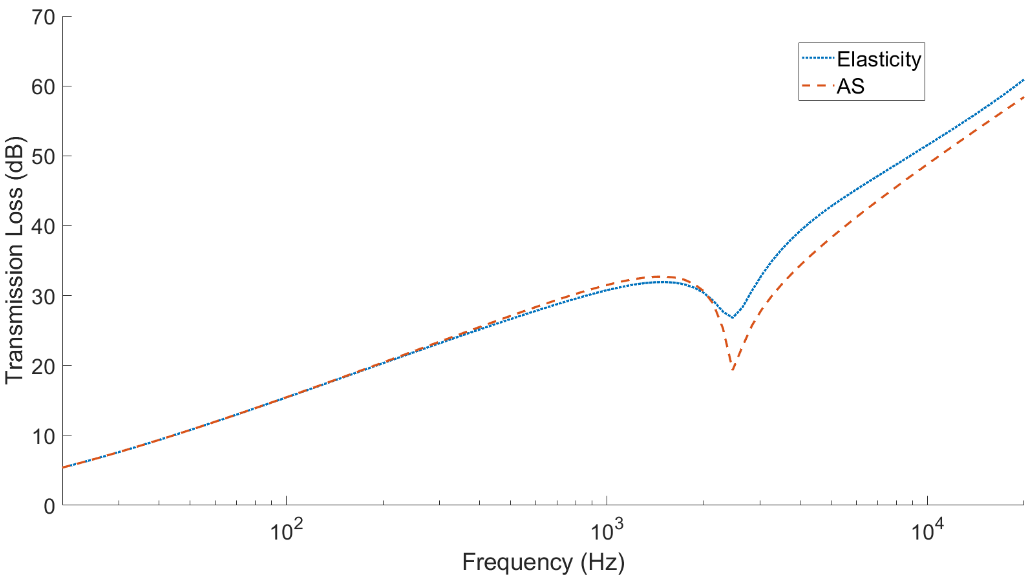

Figure 2. Transmission loss for the multi-layer plate (Plasterboard/Glue/Plasterboard) under diffuse field excitation

the frequency limit is required to safely use the AS model for the vibroacoustic problems of the multi-layer system. In this direction, as a primary step, the next section discusses about deriving the analytical frequency limit for the Love-Kirchoff plate model.

4 FREQUENCY LIMIT FOR THE LOVE-KIRCHOFF PLATE MODEL

The behaviour of the propagating waves in the framework of the thick plate model is studied in this section followed by the discussion on the analytical frequency limit for the thin plate model. The dispersion equation of thick plate can be obtained by letting $Z_{thick} = 0$.

$$k_t^4 D - m_s\omega^2 + \left(\frac{I_z m_s}{G^* h}\omega^2 - k_t^2 \frac{D m_s}{G^* h}\right)\omega^2 - k_t^2 I_z \omega^2 = 0 \tag{5}$$

The propagating wave number solution of the above dispersion relation is written as:

$$k_t = \pm\sqrt{\frac{m_s\omega^2}{2G^* h} + \frac{I_z\omega^2}{2D} + \sqrt{\frac{m_s\omega^2}{D} + \left(\frac{m_s\omega^2}{2G^* h} - \frac{I_z\omega^2}{2D}\right)^2}} \tag{6}$$

By defining $k_b^2 = \omega\sqrt{\frac{m_s}{D}}, k_s^2 = \frac{m_s\omega^2}{G^* h}$ and $k_r^2 = \frac{I_z\omega^2}{D}$, Eq.(6) is rewritten in a compact form as:

$$k_t = \pm\sqrt{\frac{1}{2}\left(k_s^2 + k_r^2 + \sqrt{4k_b^4 + (k_s^2 - k_r^2)^2}\right)} \tag{7}$$

It can be observed from Eq.(6) that at low frequencies $k_b >> k_s, k_r$, leading to the pure bending wave propagation. In this case, we have

$$k_t \approx k_b = \sqrt{\omega\sqrt{\frac{m_s}{D}}} \quad \text{(at low frequencies)} \tag{8}$$

At higher frequencies, $k_s >> k_b$, leading to pure shear wave propagation.

$$k_t \approx k_s = \omega\sqrt{\frac{m_s}{G^* h}} \quad \text{(at high frequencies)} \tag{9}$$

With this regard, k_b and k_s are referred as bending and shear wave numbers respectively in the further discussions. Based on the percentage error between the wave number pertaining to the bending wave (Eq.(8)) and the full propagating wave (Eq.(6)), the limiting frequency for the use of thin plate model for the vibroacoustic problems can be found. By defining $C = k_b/k_s$, it can be seen from the Eq.(10a) that, the ratio between k_t and k_b (thus the error,ε) is the function of only the Poisson's ratio, shear correction factor and C.

$$\frac{k_t}{k_b} = \frac{1}{2}\sqrt{\frac{2+\xi(1-\nu)}{C^2} + \sqrt{16 + \left(\frac{2-\xi(1-\nu)}{C^2}\right)^2}} \tag{10a}$$

$$\varepsilon = \left(1 - \frac{1}{k_t/k_b}\right)100\% \tag{10b}$$

The value for C can be chosen such that ε is below the accepted error percentage and the frequency limit for the thin isotropic plate model can be found as given by Eq.(11).

$$\omega_{limit} = \frac{\xi}{2C^2 h}\sqrt{\frac{12E}{\rho}\frac{1-\nu}{1+\nu}} \tag{11}$$

It can be observed that the influence of material properties on the frequency limit is small when the plate thickness is large and vice versa. To illustrate this frequency limit, the transmission loss (TL) curves for the plasterboard and concrete (Table 1) are computed from the transfer matrices of thin plate, thick plate, Guyader model and compared with linear elasticity solution for an oblique plane wave incidence with $\theta = 60^o$. The TL curves obtained for plasterboard (Fig. 3) show that the thick plate and Guyader models estimate better with elasticity solution. It is to be noted that the frequency limit is computed from the Eq.(11) by having $C = 4$, which corresponds to the error percentage (ε) of 1.94%. The frequency limit for the thin plate model in this case is found to be before the coincidence frequency. But in the case of concrete material, Fig. 4 shows that the frequency limit spotted after the coincidence frequency, which is interesting to note.

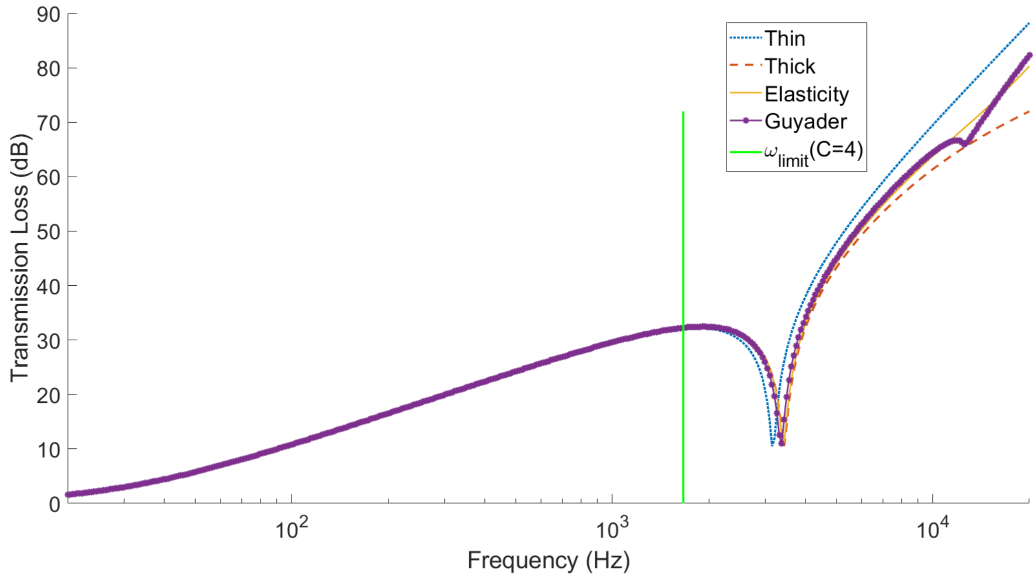

Figure 3. Transmission loss for the plasterboard under plane wave excitation with $\theta = 60^o$

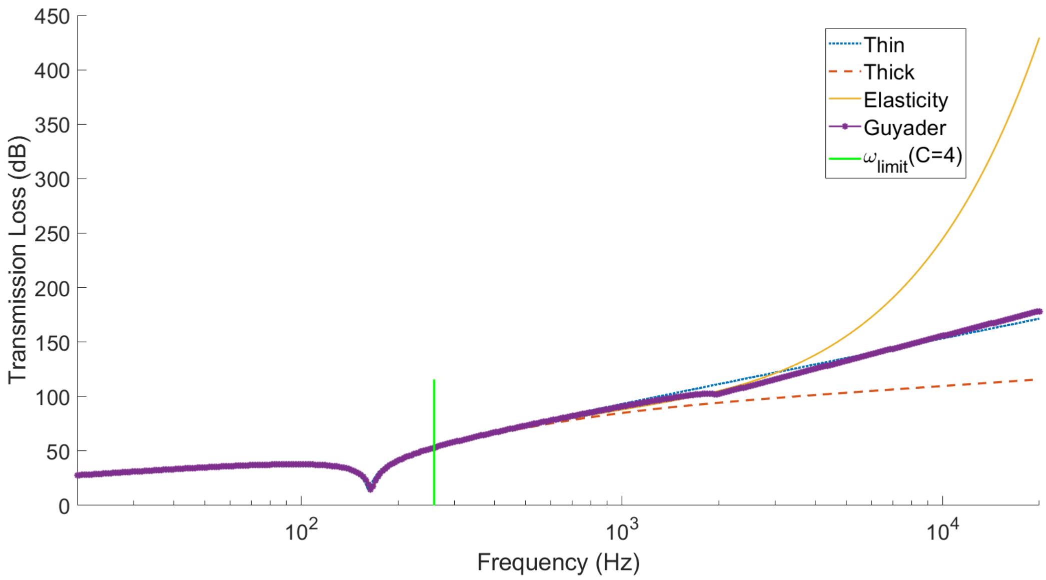

Figure 4. Transmission loss for the concrete under plane wave excitation with $\theta = 60^o$

5 COMPARISON OF PLATE MODELS FOR THE MULTI-LAYER PLATE EXAMPLE

In this section, the same multi-layer plate (discussed in section 3) is taken for the comparison of equivalent thin plate models with linear elasticity solution. From Fig. 5, it can be observed that, since Guyader and RKU models include the shear behaviour of the multi-layer plate in their formulation, they predict the critical frequency of the plate with good approximation though the equivalent properties corresponds to thin plate. But it is also seen that they exhibit poor correspondence with elasticity solution at higher frequencies, because these models do not account for compressional effect in their theories. Also, it must be noted that, as the parameters of equivalent plate models are fitted with a thin plate model, they can not be used with other models as thick plate or full elasticity.

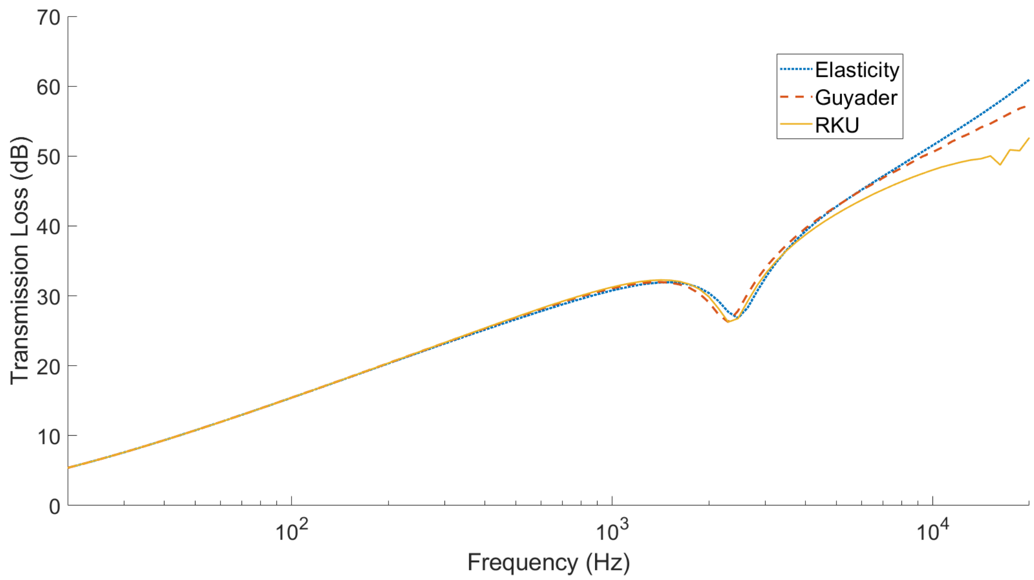

Figure 5. Transmission loss for the multi-layer plate (Plasterboard/Glue/Plasterboard) under diffuse field excitation

6 CONCLUSION

A multi-layer plate of practical application is taken to illustrate the need to find the frequency limit for the Added Stiffness (AS) model, as it is applicable only for thin multi-layer plates. From the wave propagation analysis of the thick plate model, based on the Reissner-Mindlin plate hypothesis, the analytical expression of the frequency limit for the thin plate model is derived. From the comparison plots, the deviations of transmission loss curves obtained from different models are observed after this frequency limit. It is also observed that the thick plate (Reissner-Mindlin), Guyader and RKU models provide good approximation with the linear elasticity solution by including shear effect of the plate but do not exhibit good approximation for the compressional effect of the plate.

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